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Readers' Forum

Brief discussion of previous investigations in the aerospace sciences and technical comments on papers published in the AIAA Journal *are presented in this special department. Entries must be restricted to a maximum of 1000 words, or the equivalent of one* Journal *page including formulas and figures. A discussion will be published as quickly as possible after receipt of the manuscript. Neither the AIAA nor its editors are responsible for the opinions expressed by the correspondents. Authors will be invited to reply promptly.*

Comment on "Approximate Formula of Weak Oblique Shock Wave Angle"

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Introduction

FOR flow of a supersonic ideal gas with constant specific heats over a wedge, standard analysis[1] shows the relationship between oblique shock angle β, wedge angle θ, incoming Mach number M_1, and ratio of specific heats γ is given by

$$\tan\theta = 2\cot\beta\,\frac{M_1^2\sin^2\beta - 1}{M_1^2(\gamma + \cos 2\beta) + 2} \tag{1}$$

In a recent Technical Note, Dou and Teng[2] give an analysis to show Eq. (1) may be approximated for $\theta \ll 1$ by the following:

$$\tan\beta = \frac{1}{\sqrt{M_1^2 - 1}} + \frac{\gamma + 1}{4}\,\frac{M_1^4}{(M_1^2 - 1)^2}\,\theta \tag{2}$$

However, for high values of M_1, there is an approximation that is both simpler and better. This is given by Liepmann and Roshko[1] for the distinguished limit $M_1\beta \gg 1$ and $\beta \ll 1$ (and, consequently, $\theta \ll 1$):

$$\beta = [(\gamma + 1)/2]\theta \tag{3}$$

The improved approximation (3) cannot be deduced by considering Eq. (2) in the high Mach number limit but requires a return to Eq. (1).

The relative merits of Eqs. (2) and (3) are easily seen when their predictions are compared with the exact solution of Eq. (1) as shown in Figs. 1 and 2. Figure 1 shows β vs θ, with M_1 and γ fixed at 10 and 7/5, respectively. Dou and Teng's formula [Eq. (2)] matches well near $\theta = 0$, whereas Liepmann and Roshko's [Eq. (3)] is superior at higher θ. When β is plotted vs M_1, with θ and γ held fixed at 15 deg and 7/5, respectively, Dou and Teng's approximation is better for low M_1, whereas Liepmann and Roshko's is better for high M_1. Thus, for a given *small, fixed* θ, there are Mach numbers for which Eq. (3) gives a better approximation for β than does Eq. (2). The same conclusions can be reached for Dou and Teng's extension of Eq. (2) to a quadratic expression in θ.

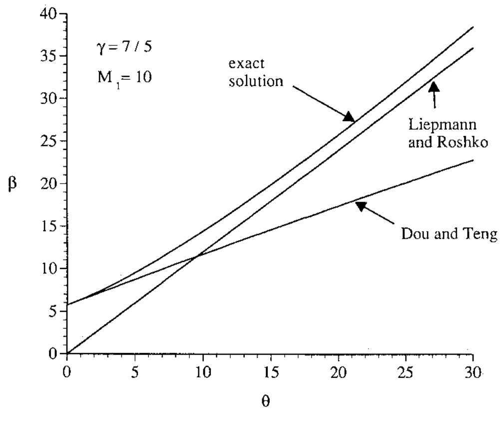

Fig. 1 Wave angle vs wedge angle for fixed incoming Mach number.

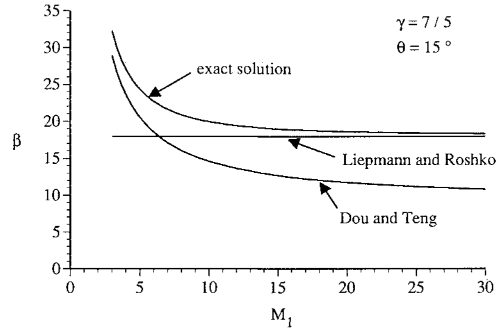

Fig. 2 Wave angle vs Mach number for fixed wedge angle.

References

[1]Liepmann, H. W., and Roshko, A., *Elements of Gasdynamics*, Wiley, New York, 1957, pp. 84–93.

[2]Dou, H.-S., and Teng, H.-Y., "Approximate Formula of Weak, Oblique Shock Wave Angle," *AIAA Journal*, Vol. 30, No. 3, 1992, pp. 837–839.

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Introduction

WE would like to thank Professor Powers for his comments on our work. The primary result of Ref. 1 is a quadratic expression in θ [Eq. (13) of Ref. 1], which was

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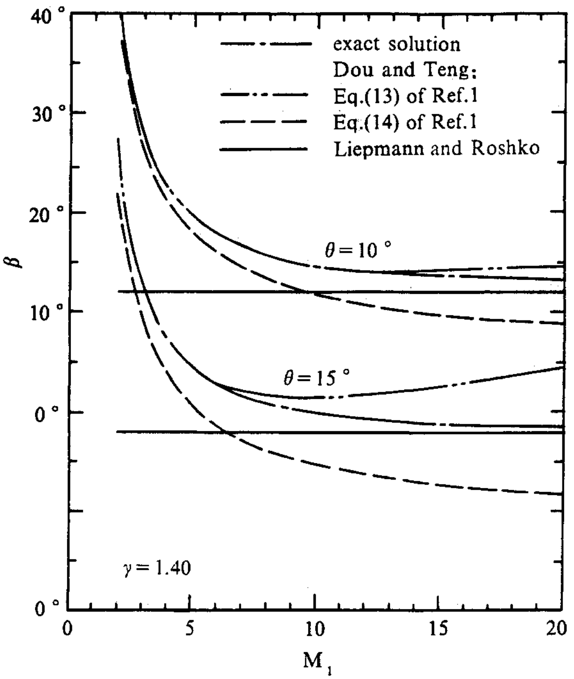

Fig. 1 Shock-wave angle vs upstream Mach number.

approximately obtained at the condition of small θ angle and is used for the ordinary M_1 range in supersonic flow. As the present authors mentioned, this formula gives a satisfactory approximation of the exact value of wave angle for a wide range of M_1 and a limited range of θ angle. For $\theta \leq 15$ deg and $2 \leq M_1 \leq 7$, the relative error of the formula to the exact value is $<3\%$. Our linear expression in θ angle [Eq. (14) of Ref. 1 or Eq. (2) of Powers' Comment] only is the further approximate form of Eq. (13) in Ref. 1 for very small θ angles. In the same range of M_1 and θ as above, the linear formula also gives a good approximation.

Powers emphasizes the superiority of Liepmann and Roshko's equation[2] [Eq. (3) of Powers' Comment] at very high M_1, which is obviously correct. The present authors did not deny the merit of Liepmann and Roshko's formula at the high M_1 condition. It should be noted that Dou and Teng's formulas were derived for a weak oblique shock wave and are only applied in the condition of small θ angle and a certain M_1 range. As we did in our paper, all equations are applicable at low M_1; e.g., $2 \leq M_1 \leq 10$, for small θ angles.[1] Therefore, one should not extend Dou and Teng's formula to high Mach numbers or large θ angles. Just as Powers described, Dou and Teng's formula is better for the low M_1 range whereas Liepmann and Roshko's formula is better for the high M_1 range. The calculated results with these formulas are compared with the exact solution in Fig. 1. In addition, another exact solution of wave angle β is given in Ref. 3, which also expresses β as an explicit function of M_1 and θ.

In conclusion, we still suggest that Eq. (13) of Ref. 1 is a good approximation of wave angle over wide ranges of M_1 and θ. Its linear equation is only applicable for very small θ angles. Liepmann and Roshko's formula can display its advantage only at the very high M_1 condition.

References

[1]Dou, H.-S., and Teng, H.-Y., "Approximate Formula of Weak Oblique Shock Wave Angle," *AIAA Journal,* Vol. 30, No. 3, 1992, pp. 837–839.

[2]Liepmann, H. W., and Roshko, A., *Elements of Gasdynamics,* Wiley, New York, 1957, pp. 84–93.

[3]Mascitti, V. R., "A Closed-Form Solution to Oblique Shock-Wave Properties," *Journal of Aircraft,* Vol. 6, No. 1, 1969, p. 66.